Let E be an elliptic curve, without complex multiplication and defined over some number field F. To each finite place v of F where E has good reduction there correspond two conjugate eigenvalues of Frobenius, and one of those has argument $\theta_v \in [0, \pi]$. The Sato-Tate conjecture predicts that the θ_v are equidistributed in $[0, \pi]$ with respect to the Sato-Tate measure $\mu = \frac{2}{\pi} \sin^2 \theta \, d\theta$. This conjecture is now proved when F is totally real and under the additional assumption that there exists a finite place of F where E has multiplicative reduction.

The proof uses automorphic methods and results from three papers, the first one from Clozel, Harris and Taylor, the second one from Harris, Shepherd-Barron, Taylor, and finally the last and more recent one by Taylor alone. In the first and last paper one extends the Taylor-Wiles methods (enriched with some entierely new ideas) to the case of unitary groups. In the middle one one uses a particular family of projective hypersurfaces, whose mod. ℓ cohomologies constitute a wide collection of symplectic modulo ℓ Galois representations.

In my four lectures I shall explain the main features of the proof.